Pre-Calculus Honors Summer Assignment

Dear Future Pre-Calculus Honors Student,

Congratulations on your successful completion of Algebra 2! Below you will find the summer assignment questions. It is assumed that these concepts, along with many others, have been mastered by you, the incoming Pre-Calculus Honors student. There will be an assessment on this material within the first five days of school. This assessment will be an indicator of your foundation for the course, hence your success with the new material Pre-Calculus Honors offers.

It is **strongly** suggested that you take this assignment seriously. This assignment should be completed prior to the first day of school. Waiting to start it the night before the first day of school is not a wise idea. When you find yourself unable to answer a question, do not skip it – research it. That research can be in the form of a parent, a friend, free on-line help like brightstorm.com and khanacademy.org, or your old Algebra 2 notebook. The websites are user-friendly and offer excellent explanations. Definitely check them out as a resource.

When you return to school in September, we expect that you will have gone through all of the questions. Feel free to ask specific questions pertaining to the summer assignment within the first few days of school. The Pre-Calculus teachers are here to help.

Enjoy your summer and we look forward to meeting you in September.

Name ______________________________ Date ______________

Pre-Calculus Honors **Summer Assignment**

Directions: Please do all work on a separate sheet of paper. You may draw the graphs on the coordinate planes that appear within this packet. It is strongly recommend that these problems be completed without the aid of a calculator, unless you are directed to use one. You will not be permitted to use a calculator on many of the Pre-Calculus Honors assessments throughout the year, so this gives you an opportunity to sharpen your mental math skills. There will be an assessment on this material within the first 5 days of school. Good luck!

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

1) slope 4, passes through $(6, 9)$
2) passes through $(8, 5)$ and $(3, -10)$
3) passes through $(-6, -6)$, parallel to $4x - 3y = -24$
4) passes through $(4, 2)$, perpendicular to $y = -2x + 3$

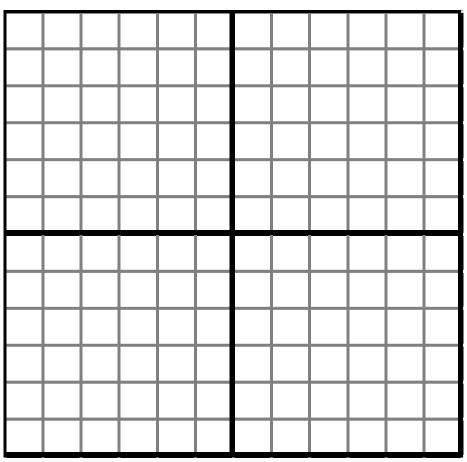
Find the x-intercept and the y-intercept of each equation.

5) $2x - 3y + 8 = 0$

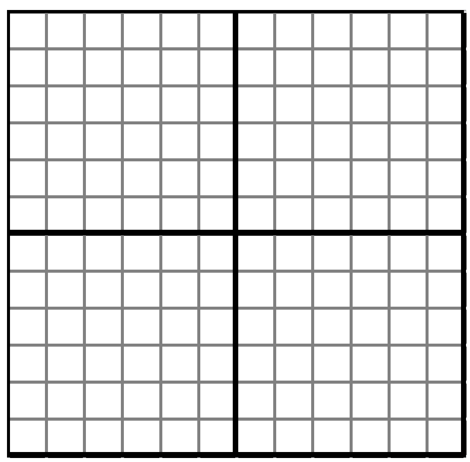
6) $f(x) = 4x - 10$

Graph each linear equation.

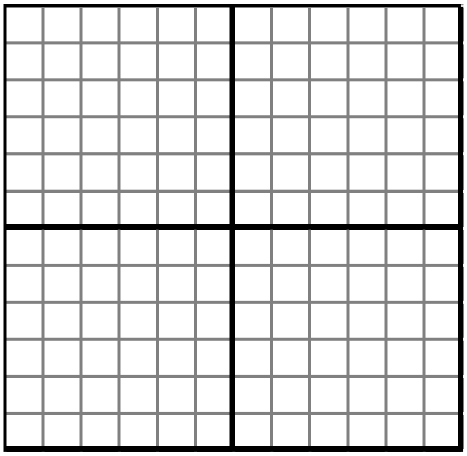
7) $y = \frac{1}{2}x - 3$



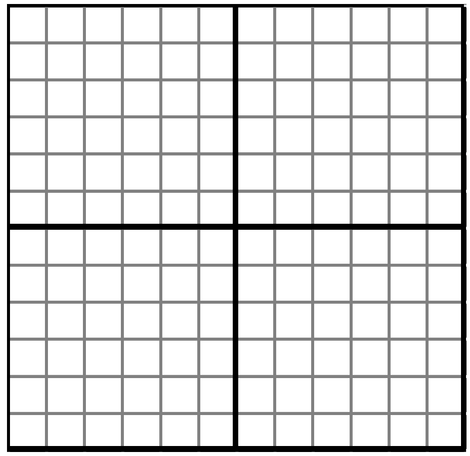
8) $y = 2$



9) $f(x) = -\frac{3}{2}x + 5$

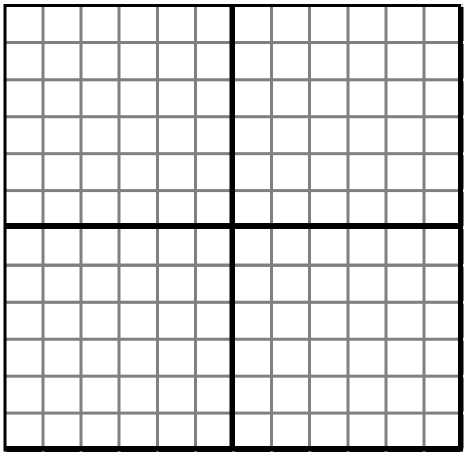


10) $x + 3y = -6$

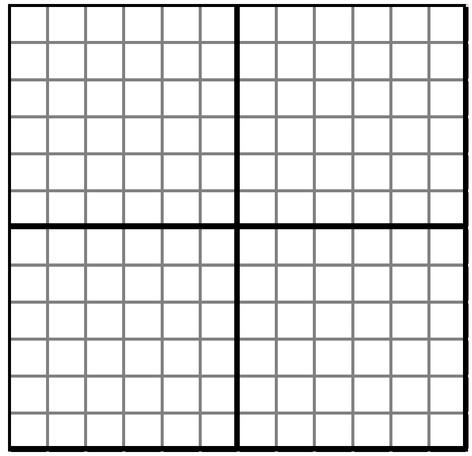


Graph each linear inequality.

11) $y > 3x + 4$



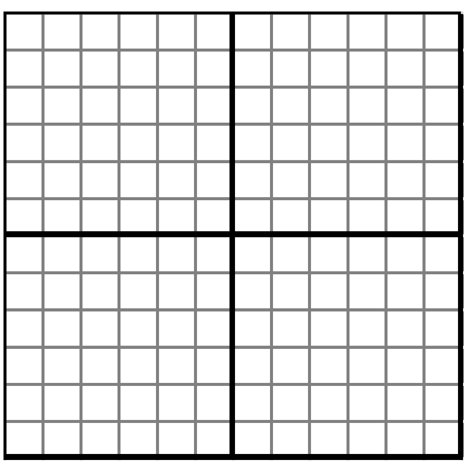
12) $10 - 5y \geq 2x$



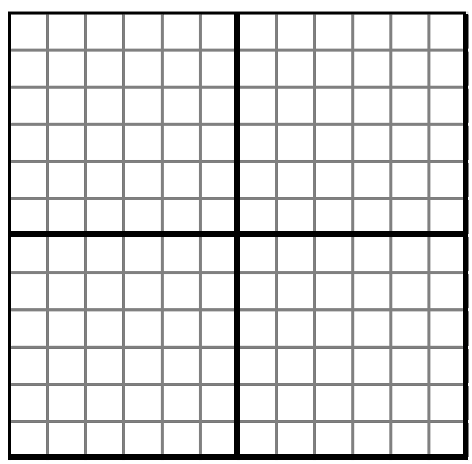
Solve each system by graphing. Describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

Type of System	The Graph	Solution
consistent and independent	intersecting lines	one solution, (x, y)
consistent and dependent	coinciding lines	infinitely many solutions
inconsistent	parallel lines	no solution

13) $\begin{cases} x = 3 \\ y = -\frac{2}{3}x - 1 \end{cases}$



14) $\begin{cases} 2x + y = 5 \\ 8x + 4 = -4y \end{cases}$



Solve each system algebraically (substitution or elimination).

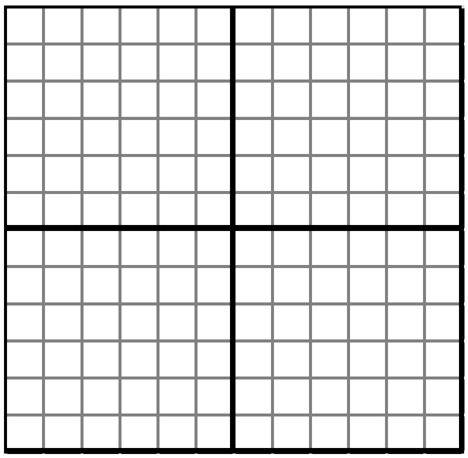
15) $\begin{cases} y + 16 = 5x \\ 2x + 3y = 3 \end{cases}$

16) $\begin{cases} 2x - 3y = 1 \\ 4x - 5y = 7 \end{cases}$

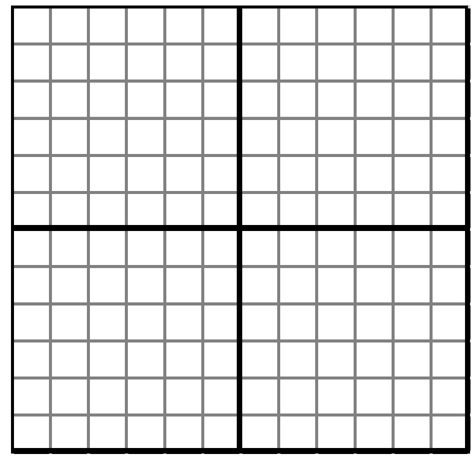
17) $\begin{cases} 5x - 2y = 12 \\ -\frac{5}{4}x + \frac{1}{2}y = -3 \end{cases}$

Solve the system of inequalities by graphing.

18) $\begin{cases} y < -3x - 1 \\ 3x - 6y > 6 \end{cases}$



19) $\begin{cases} y \le \frac{1}{4}x + 2 \\ y \ge \frac{3}{2}x - 3 \\ x + y \ge 2 \end{cases}$



Simplify each expression using the laws of exponents. No negative exponents in your final answer.

20) $\left(-2x^3y^2\right)^5$

21) $\left(2x^{-3}y^3\right)\left(-7x^5y^{-6}\right)$

22) $\left(\dfrac{3a^3}{b^4}\right)^{-2}$

23) $\dfrac{15c^5d^3}{-18c^2d^7}$

Simplify each expression.

24) $(3m+4)(2m-5)$

25) $(2x+5y)^2$

26) $(2x+5)\left(4x^2-10x+25\right)$

27) $5c\left(2c^2-3c+4\right)-2c(7c-8)$

28) $\dfrac{4a^3b-6ab+2ab^2}{2ab}$

29) $(3x-2)^3$

Simplify each radical expression. No decimal approximations!

30) $-\sqrt{36x^8y^2}$

31) $\sqrt[3]{27b^{18}c^{11}}$

32) $\sqrt[4]{81(x+4)^4}$

33) $\sqrt{75}$

34) $\sqrt{32}$

35) $\sqrt[3]{40}$

36) $\sqrt{12c^6d^5}$

37) $\sqrt[4]{8x^3y^2} \bullet \sqrt[4]{2x^5y^2}$

38) $2\sqrt{3}-7\sqrt{3}+6\sqrt{2}$

39) $4\sqrt{8}+3\sqrt{50}$

40) $\sqrt[3]{\dfrac{32x^6}{4}}$

41) $(7-\sqrt{3})(7+\sqrt{3})$

Rationalize each denominator. (Use conjugates to rationalize binomial denominators.)

42) $\dfrac{2}{\sqrt{3}}$

43) $\dfrac{15}{\sqrt{5}}$

44) $\dfrac{2}{\sqrt{5}-1}$

45) $\dfrac{9-2\sqrt{3}}{\sqrt{3}+6}$

Evaluate each expression involving rational exponents.

46) $49^{1/2}$

47) $125^{-1/3}$

48) $27^{2/3}$

49) $16^{5/4}$

Perform each operation if $f(x)=3x-7$ and $g(x)=x^2+3$.

50) $f(20)$

51) $g(a-4)$

52) $g[f(-1)]$

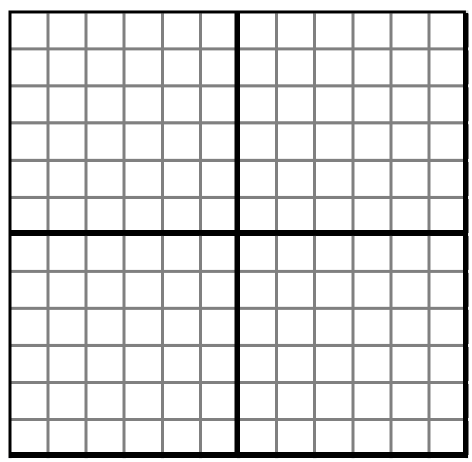
53) $f[g(x)]$

Graph each quadratic function and complete the chart.

Reminder: The formula for the equation of the axis of symmetry is $x=-\frac{b}{2a}$

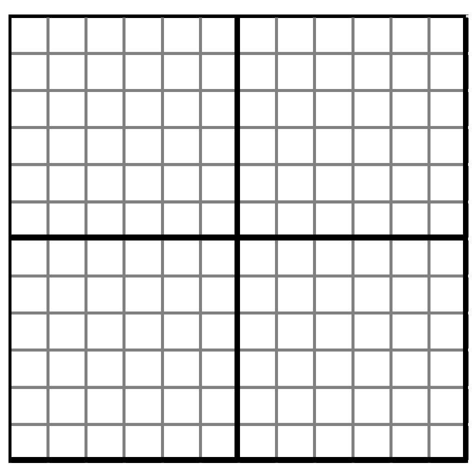
54) $f(x)=x^2+2x-3$

opens up or down?	
y-intercept	
Axis of Symmetry	
Vertex	(,)
Domain	
Range	



55) $f(x)=-2x^2+8x-5$

opens up or down?	
y-intercept	
Axis of Symmetry	
Vertex	(,)
Domain	
Range	



Mr. Goodman, who knows a lot about physics, is going to fire a rocket to start off this year's Battle of the Classes. He has done testing on the rocket to ensure it will not hit any students. If the rocket is launched upwards with an initial velocity of 120 feet per second, its height $h(t)$ (in feet) can be found by the function $h(t) = -16t^2 + 120t$, where t is the number of seconds since it was launched.

56) How long will it take for the rocket to reach its highest point?

57) What is the maximum height that the rocket will reach?

Guidelines for Factoring:

- Always look for a GCF before doing anything else.
- Consider the number of terms in the polynomial.

  Two Terms: Try factoring as a difference of two squares, or a sum or difference of two cubes.

  $$a^2 - b^2 = (a+b)(a-b)$$
  $$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
  $$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

  Three Terms: Trinomials of the form $ax^2 + bx + c$ can be factored into the product of two binomials. (Use a method that your Algebra 2 teacher taught you.)

  More Than Three Terms: Try "factoring by grouping".

- Make sure the polynomial is factored completely. This means that each remaining factor is prime.

Factor each polynomial completely.

58) $8ab^2 - 4ab$

59) $x^2 + 4x - 21$

60) $25x^2 + 10xy + y^2$

61) $9c^2 - 49d^2$

62) $3x^3 - 3x^2 - 90x$

63) $3x^2 + 28x + 32$

64) $x^3 + 5x^2 - 2x - 10$

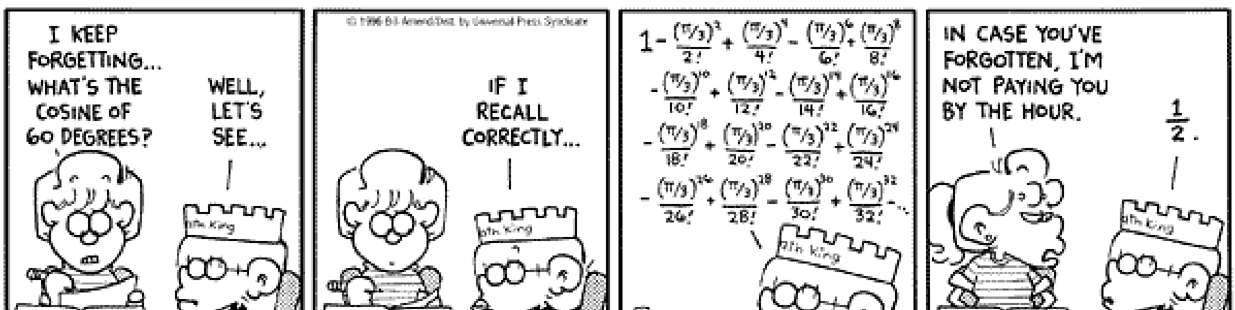
65) $6x^2 + 7x - 3$

66) $16x^3y - 81xy$

67) $x^4 - 4x^2 - 45$

68) $8x^3 + 125$

69) $18a^2 - 31ab + 6b^2$



The imaginary unit ***i*** is defined as the principal square root of -1 and can be written as $i = \sqrt{-1}$.
Since $i = \sqrt{-1}$, it naturally follows that $(i)^2 = \left(\sqrt{-1}\right)^2 \rightarrow i^2 = -1$

Higher powers of ***i*** can be found using the following method.

$$i^1 = i$$
$$i^2 = -1$$
$$i^3 = \left(i^2\right)\left(i\right) = -i$$
$$i^4 = \left(i^2\right)\left(i^2\right) = 1$$

Here are the first eight powers of ***i***.

i^1	i	i^5	i
i^2	-1	i^6	-1
i^3	$-i$	i^7	$-i$
i^4	1	i^8	1

A **complex number** is a number that can be written in $a + bi$ form.

Simplify each expression.

70) i^{14}

71) $\sqrt{-64}$

72) $\sqrt{-24}$

73) $(-3-i)-(4-5i)$

74) $(5+2i)^2$

75) $\dfrac{1+2i}{2-3i}$

Solve each equation by factoring. Express your solutions in simplified radical form, if necessary.

76) $20x^2 - 11x - 3 = 0$

77) $2x^3 - 12x^2 = -18x$

78) $x^4 = 4 - 3x^2$

Solve each equation by completing the square. Answer in simplified radical form, if necessary.

79) $x^2 + 10x + 41 = 0$

80) $x^2 + 8x + 4 = 0$

Solve each equation by using the Quadratic Formula. Answer in simplified radical form, if necessary.

81) $2x^2 + 6x - 3 = 0$

82) $x^2 + 8 = 6x - 5$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Solve each quadratic equation using any method. Answer in simplified radical form, if necessary.

83) $(x+5)(x-3) = 33$

84) $x^2 - 4x + 7 = 0$

85) Normal systolic blood pressure is a function of age. For a woman, the normal systolic blood pressure (in millimeters of mercury) is given by the function $P = .01x^2 + .05x + 107$, where x

is the woman's age. Use this function to find the age of a woman whose systolic blood pressure is measured to be 121 millimeters of mercury. (Hint: Use the Quadratic Formula)

Simplify each expression.

86) $\dfrac{2k^2 - k - 15}{k^2 - 13k + 30}$

87) $\dfrac{-2u^3 y}{15xz^5} \cdot \dfrac{25x^3}{14u^2 y^2}$

88) $\dfrac{\frac{2x-14}{8x}}{\frac{x^2-49}{4x}}$

89) $\dfrac{5}{6ab} - \dfrac{7}{8a^2}$

90) $\dfrac{5}{x+3} + \dfrac{2}{x+7}$

91) $\dfrac{4}{3x+6} - \dfrac{x+1}{x^2-4}$

92) $\dfrac{1 + \frac{y}{x}}{\frac{1}{y} + \frac{1}{x}}$

Match each parent graph to its equation.

A) $f(x) = \frac{1}{x}$

B) $f(x) = |x|$

C) $f(x) = \sqrt{x}$

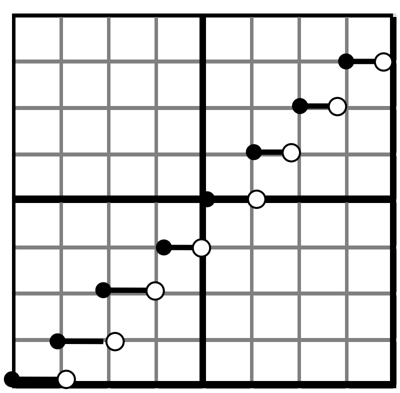
D) $f(x) = x^2$

E) $f(x) = 2^x$

F) $f(x) = \log_2 x$

G) $f(x) = x^3$

H) $f(x) = [\![x]\!]$

93)


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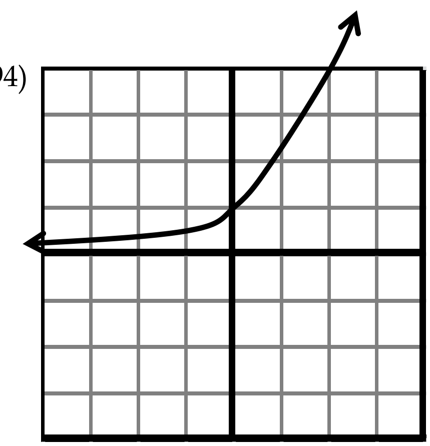

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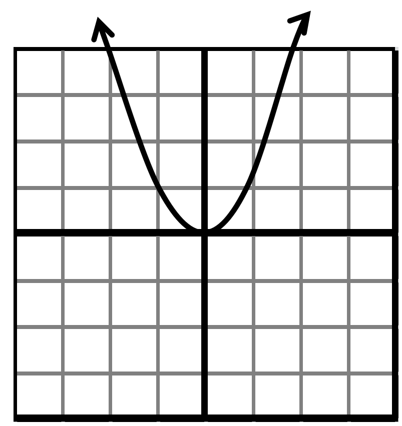

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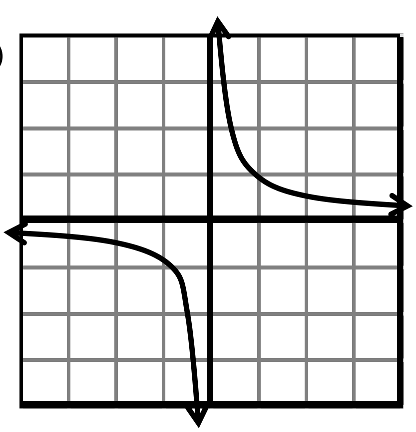

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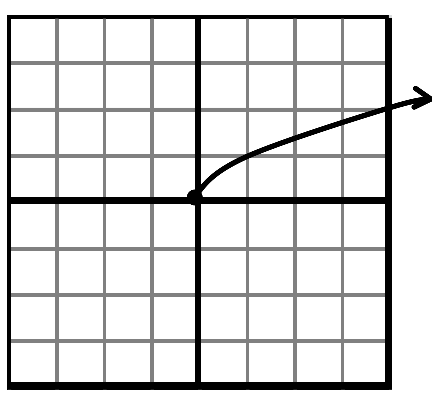

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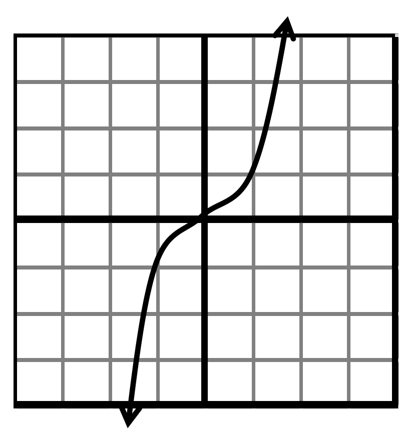

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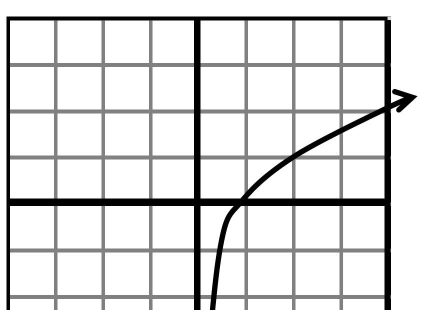

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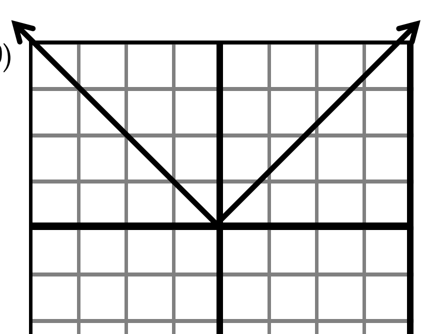


Read the following sections to help you answer some of the remaining questions:

1 Set Notation

A **set** is a collection of objects. Each object in a set is called an **element**. A set is named using a capital letter and is written with its elements listed within braces { }.

Set Name	Description of Set	Set Notation
C	pages in a chapter of a book	$C = \{35, 36, 37, 38, 39, 40\}$
A	students who made an A on the test	$A =$ {Olinda, Mario, Karen}
L	the letters from A to H	$L = \{A, B, C, D, E, F, G, H\}$
N	positive odd numbers	$N = \{1, 3, 5, 7, 9, 11, 13, \ldots\}$

To write that Olinda is *an element* of set A, write Olinda $\in A$.

Example 1 Use Set Notation

Use set notation to write the elements of each set. Then determine whether the statement about the set is *true* or *false*.

a. N is the set of whole numbers greater than 12 and less than 16. $15 \in N$

The elements in this set are 13, 14, and 15, so $N = \{13, 14, 15\}$. Because 15 is an element of N, $15 \in N$ is a true statement.

b. V is the set of vowels. g $\in V$

The elements in this set are the letters a, e, i, o, and u, so $V = \{a, e, i, o, u\}$. Because the letter g is not an element of V, a correct statement is g $\notin V$. Therefore, g $\in V$ is a false statement.

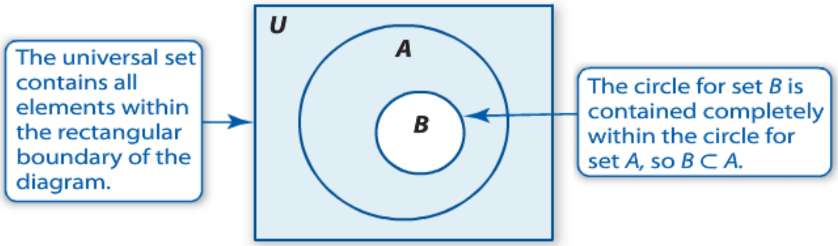
c. M is the set of months that begin with J. April $\in M$

The elements in this set are the months January, June, and July, so $M =$ {January, June, July}. Because the month of April is not an element of this set, a correct statement is April $\notin M$. Therefore, April $\in M$ is a false statement.

d. X is the set of numbers on a die. $4 \in X$

The elements in this set are 1, 2, 3, 4, 5, and 6, so $X = \{1, 2, 3, 4, 5, 6\}$. Because 4 is an element of X, $4 \in X$ is a true statement.

If every element of set B is also contained in set A, then B is called a **subset** of A, and is written as $B \subset A$. The **universal set** U is the set of all possible elements for a situation. All other sets in this situation are subsets of the universal set.



Suppose $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 2, 3\}$. Because $1 \in A$, $2 \in A$, and $3 \in A$, $B \subset A$.

The set of elements in U that are not elements of set B is called the **complement** of B, and is written as B'. In the Venn diagram, the complement of B is all of the shaded regions.

Example 2 Identify Subsets and Complements of Sets

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 4, 5, 7, 8, 9\}$, $B = \{5, 7\}$, $C = \{1, 5, 7, 8\}$, $D = \{2, 3\}$, and $E = \{6, 3\}$.

a. State whether $B \subset A$ is *true* or *false*.

$B = \{5, 7\}$ $\qquad$ $A = \{1, 4, 5, 7, 8, 9\}$

True; $5 \in A$ and $7 \in A$, so all of the elements of B are also elements of A. Therefore, B is a subset of A.

b. State whether $E \subset D$ is *true* or *false*.

$E = \{6, 3\}$ $\qquad$ $D = \{2, 3\}$

False; $6 \notin D$, so not all of the elements of E are in D. Therefore, E is not a subset of D.

c. Find A'.

Identify the elements of U that are not in A.

$A = \{1, 4, 5, 7, 8, 9\}$ $\qquad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $A' = \{0, 2, 3, 6\}$.

d. Find D'.

Identify the elements of U that are not in D.

$D = \{2, 3\}$ $\qquad$ $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, $D' = \{0, 1, 4, 5, 6, 7, 8, 9\}$.

2 Unions and Intersections

The **union** of sets A and B, written $A \cup B$, is a new set consisting of all of the elements that are in either A or B. The **intersection** of sets A and B, written $A \cap B$, is a new set consisting of elements found in A and B. If two sets have no elements in common, their intersection is called the **empty set**, and is written as $\varnothing$ or $\{\,\}$.

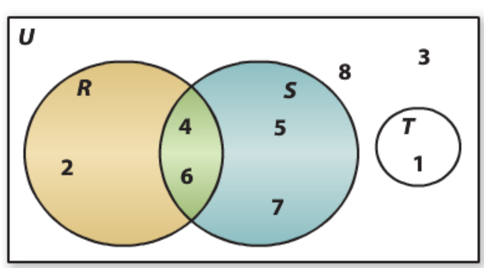
Example 3 Find the Union and Intersection of Two Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $R = \{2, 4, 6\}$, $S = \{4, 5, 6, 7\}$, and $T = \{1\}$.

a. Find $R \cup S$.

The union of R and S is the set of all elements that belong to R, S, or to both sets.

So, $R \cup S = \{2, 4, 5, 6, 7\}$.



b. Find $R \cap S$.

The intersection of R and S is the set of all elements found in both R and S.

So, $R \cap S = \{4, 6\}$.

1 Describe and Analyze Matrices

A **matrix** is a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets. Each value in the matrix is called an **element**. A matrix is usually named using an uppercase letter.

The element −1 is in Row 2, Column 1, depicted by a_{21}.

$$A = \underbrace{\begin{bmatrix} 8 & -2 & 5 & 6 \\ -1 & 3 & -3 & 6 \\ 7 & -8 & 1 & 4 \end{bmatrix}}_{\text{4 columns}} \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\} \text{3 rows}$$

The element −8 is in Row 3, Column 2, depicted by a_{32}.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix, which is read *m by n*. Matrix A above is a 3×4 matrix because it has 3 rows and 4 columns.

Example 1 Dimensions and Elements of a Matrix

Use $A = \begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix}$ to answer the following.

a. State the dimensions of A.

$$\underbrace{\begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix}}_{\text{3 columns}} \left.\vphantom{\begin{matrix}a\\b\end{matrix}}\right\} \text{2 rows}$$

Because A has 2 rows and 3 columns, the dimensions of A are 2×3.

b. Find the value of a_{13}.

$$\begin{array}{c} \text{Column 3} \\ \downarrow \end{array}$$

$$\begin{bmatrix} 4 & 9 & -18 \\ -2 & 11 & 3 \end{bmatrix} \leftarrow \text{Row 1}$$

Because a_{13} is the element in row 1, column 3, the value of a_{13} is −18.

Certain matrices have special names. For example, a matrix that has one row is called a **row matrix**, and a matrix with one column is a **column matrix**. A matrix that has the same number of rows and columns is known as a **square matrix**, and a matrix in which every element is zero is called a **zero matrix**.

Row Matrix

$$\begin{bmatrix} 8 & -5 & 2 & 4 \end{bmatrix}$$

Column Matrix

$$\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Square Matrix

$$\begin{bmatrix} -4 & 2 \\ 3 & 9 \end{bmatrix}$$

Zero Matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Two matrices are **equal matrices** if and only if each element of one matrix is equal to the corresponding element in the other matrix. So, matrix A and B shown below are equal matrices.

$$A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

Notice that for two matrices to be equal, they must have the same number of rows and columns.

2 Matrix Operations

Matrices can be added or subtracted if and only if they have the same dimensions.

KeyConcept Adding and Subtracting Matrices

To add or subtract two matrices with the same dimensions, add or subtract their corresponding elements.

$$\begin{array}{ccccc} A & + & B & = & A+B \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & + & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} \end{array}$$

$$\begin{array}{ccccc} A & - & B & = & A-B \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & - & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix} \end{array}$$

Example 2 Add and Subtract Matrices

Find each of the following for $A = \begin{bmatrix} 8 & 3 \\ -5 & 14 \end{bmatrix}$, $B = \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix}$, and $C = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$.

a. $A + B$

$$A + B = \begin{bmatrix} 8 & 3 \\ -5 & 14 \end{bmatrix} + \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix} \qquad \text{Substitution}$$

$$= \begin{bmatrix} 8+12 & 3+(-7) \\ -5+6 & 14+(-23) \end{bmatrix} \text{ or } \begin{bmatrix} 20 & -4 \\ 1 & -9 \end{bmatrix} \qquad \text{Add corresponding elements.}$$

b. $B - C$

$$B - C = \begin{bmatrix} 12 & -7 \\ 6 & -23 \end{bmatrix} - \begin{bmatrix} 2 \\ 9 \end{bmatrix} \qquad \text{Substitution}$$

B is a 2×2 matrix and C is a 2×1 matrix. Since these dimensions are not the same, you cannot subtract the matrices.

You can multiply any matrix by a constant called a **scalar**. When you do this, you multiply each individual element by the value of the scalar.

Example 3 Scalar Multiplication

Find each product.

a. $3\begin{bmatrix} -6 & -3 & 7 \\ 10 & 2 & -15 \end{bmatrix}$

$$3\begin{bmatrix} -6 & -3 & 7 \\ 10 & 2 & -15 \end{bmatrix} = \begin{bmatrix} 3(-6) & 3(-3) & 3(7) \\ 3(10) & 3(2) & 3(-15) \end{bmatrix} \text{ or } \begin{bmatrix} -18 & -9 & 21 \\ 30 & 6 & -45 \end{bmatrix}$$

b. $-4\begin{bmatrix} 2 & -9 \\ 7 & 3 \\ -11 & 4 \end{bmatrix}$

$$-4\begin{bmatrix} 2 & -9 \\ 7 & 3 \\ -11 & 4 \end{bmatrix} = \begin{bmatrix} -4(2) & -4(-9) \\ -4(7) & -4(3) \\ -4(-11) & -4(4) \end{bmatrix} \text{ or } \begin{bmatrix} -8 & 36 \\ -28 & -12 \\ 44 & -16 \end{bmatrix}$$

Many properties of real numbers also hold true for matrices. A summary of these properties is listed below.

KeyConcept Properties of Matrix Operations

For any matrices A, B, and C for which the matrix sum and product are defined and any scalar k, the following properties are true.

Commutative Property of Addition	$A + B = B + A$
Associative Property of Addition	$(A + B) + C = A + (B + C)$
Left Scalar Distributive Property	$k(A + B) = kA + kB$
Right Scalar Distributive Property	$(A + B)k = kA + kB$

Multi-step operations can be performed on matrices. The order of these operations is the same as with real numbers.

Example 4 Multi-Step Operations

Find $4(P + Q)$ if $P = \begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix}$ and $Q = \begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}$.

$$4(P+Q) = 4\left(\begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix} + \begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}\right)$$ Substitution

$$= 4\begin{bmatrix} 3 & 8 & -2 \\ -5 & 5 & -4 \end{bmatrix} + 4\begin{bmatrix} -4 & 5 & 7 \\ 3 & -10 & -6 \end{bmatrix}$$ Distributive Property

$$= \begin{bmatrix} 12 & 32 & -8 \\ -20 & 20 & -16 \end{bmatrix} + \begin{bmatrix} -16 & 20 & 28 \\ 12 & -40 & -24 \end{bmatrix}$$ Multiply by the scalar.

$$= \begin{bmatrix} 12 + (-16) & 32 + 20 & -8 + 28 \\ -20 + 12 & 20 + (-40) & -16 + (-24) \end{bmatrix}$$ Add.

$$= \begin{bmatrix} -4 & 52 & 20 \\ -8 & -20 & -40 \end{bmatrix}$$ Simplify.

You can use the same algebraic methods for solving equations with real numbers to solve equations with matrices.

Example 5 Solving a Matrix Equation

Given $A = \begin{bmatrix} -9 & 15 & 4 \\ 2 & -10 & -5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 5 & -7 & 8 \\ 14 & 10 & -3 \end{bmatrix}$**, solve** $4X - B = A$ **for** X**.**

$4X - B = A$ — Original equation

$4X = A + B$ — Add B to each side.

$X = \frac{1}{4}(A + B)$ — Divide each side by 4.

$X = \frac{1}{4}\left(\begin{bmatrix} -9 & 15 & 4 \\ 2 & -10 & -5 \end{bmatrix} + \begin{bmatrix} 5 & -7 & 8 \\ 14 & 10 & -3 \end{bmatrix}\right)$ — Substitution

$X = \frac{1}{4}\begin{bmatrix} -4 & 8 & 12 \\ 16 & 0 & -8 \end{bmatrix}$ — Add.

$X = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & -2 \end{bmatrix}$ — Multiply by the scalar.

Matrix equations can be used in real-world situations.

Real-World Example 6 Use a Matrix Equation

CELL PHONES **Allison took a survey of her high school to see which class sent the most text messages, pictures, and talked for the most minutes on their cell phones each week. The averages for the freshmen, sophomores, juniors, and seniors are shown.**

Class	Texts	Pictures	Calls
freshman	20	3	163
sophomore	25	4	170
junior	15	7	178
senior	22	3	190

a. If each text message costs \$0.10, each picture costs \$0.75, and each minute on the phone costs \$0.05, find the average weekly cell phone costs for each class. Express your answer as a matrix.

Step 1 Write a matrix equation for the total cost X. Let T represent the number of texts for all classes, P represent the number of pictures, and C represent the number of call minutes.

$X = 0.10T + 0.75P + 0.05C$

Step 2 Solve the equation.

$X = 0.10T + 0.75P + 0.05C$ — Original equation

$= 0.10\begin{bmatrix} 20 \\ 25 \\ 15 \\ 22 \end{bmatrix} + 0.75\begin{bmatrix} 3 \\ 4 \\ 7 \\ 3 \end{bmatrix} + 0.05\begin{bmatrix} 163 \\ 170 \\ 178 \\ 190 \end{bmatrix}$ — Substitution

$$= \begin{bmatrix} 2.00 \\ 2.50 \\ 1.50 \\ 2.20 \end{bmatrix} + \begin{bmatrix} 2.25 \\ 3.00 \\ 5.25 \\ 2.25 \end{bmatrix} + \begin{bmatrix} 8.15 \\ 8.50 \\ 8.90 \\ 9.50 \end{bmatrix} \text{ or } \begin{bmatrix} 12.40 \\ 14.00 \\ 15.65 \\ 13.95 \end{bmatrix}$$ Multiply by the scalars.

The final matrix indicates average weekly cell phone costs for each class. Therefore, on average, each freshman spent \$12.40, each sophomore spent \$14.00, each junior spent \$15.65, and each senior spent \$13.95.

b. If there are 100 freshmen, 180 sophomores, 250 juniors, and 300 seniors that use cell phones at Allison's school, use her survey results to estimate the total number of text messages sent, pictures sent, and minutes used on the cell phone each week by these students. Express your answer as a matrix.

Step 1 Write a matrix equation for the total usage X. Let F represent freshmen, S represent sophomores, J represent juniors, and N represent seniors.

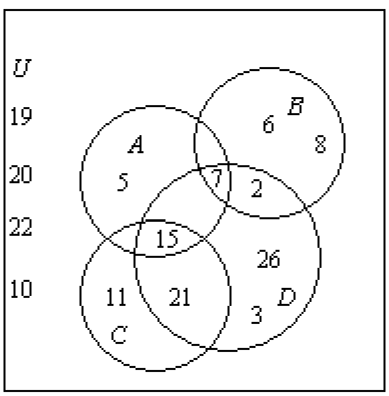
$X = 100F + 180S + 250J + 300N$

Step 2 Solve the equation.

$$X = 100F + 180S + 250J + 300N$$
$$= 100[20 \quad 3 \quad 163] + 180[25 \quad 4 \quad 170] + 250[15 \quad 7 \quad 178] + 300[22 \quad 3 \quad 190]$$
$$= [16{,}850 \quad 3670 \quad 148{,}400]$$

The final matrix indicates the average weekly totals for each type of cell phone use. Therefore, there were 16,850 texts, 3670 pictures, and 148,400 minutes used by these students.

101. Use the Venn diagram to find $A \cup B$, $C \cap D$, and $(A \cup C) \cup D$.



102. Simplify the expression $\dfrac{3 - 9i\sqrt{5}}{3 + 2i\sqrt{5}}$ by using complex conjugates to write quotients of complex numbers in standard form.

103. In an alternating-current circuit, the voltage E is given by $E = IZ$, where I is the current (in A) and Z is the impedance (in Ω). Each of these can be represented by complex numbers. Find the complex number representation for E if $I = 0.835 - 0.427i$ amperes and $Z = 250 + 170i$ ohms.

104. N is the set of odd natural numbers greater than 3 and less than 13. Use set notation to write the elements of N.

105. Simplify the expression. $\dfrac{x^{\frac{4}{7}} \bullet x^{\frac{3}{7}}}{x^{\frac{1}{7}}}$

106. Find $\begin{bmatrix} 5 & 9 \\ -3 & -9 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -2 & -6 \end{bmatrix}$.

107. Find $\begin{bmatrix} 4 & 5 \\ -3 & -6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & -4 \end{bmatrix}$.

108. If $A = \begin{bmatrix} -3 & 7 & -5 \\ 3 & 7 & 9 \end{bmatrix}$, find –4*A*.

109. Find each of the following for $A = \begin{bmatrix} 7 & 5 & 4 \\ 2 & 8 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 2 & 9 \\ 12 & 8 & 4 \end{bmatrix}$.

a. $A + B$
b. $5A - B$

110. Solve the system of equations.

$-7x + 2y - 3z = -30$
$-x + 4y - z = -4$
$6x + y + z = 34$

111. Describe the set of numbers using interval notation.

$8 \geq x > 6$

112. Describe the set of numbers using set-builder notation.

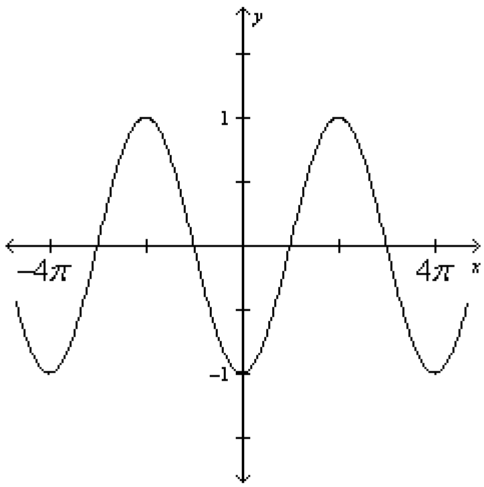
$x \geq 4$

113. Describe the set of numbers using set-builder notation.

{9, 10, 11, 12, 13, ...}

114. Determine the domain of the function $h(x) = \dfrac{5x}{x\left(x^2 - 49\right)}$.

Use the graph to determine the domain and range of the relation, and state whether the relation is a function.

115. 

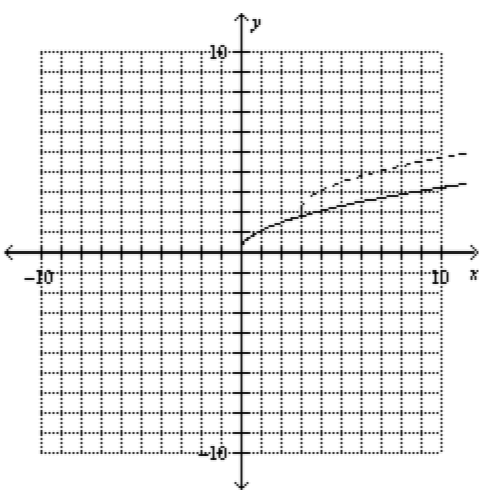
116. Find $f(m - 2)$ for $f(x) = 2x^2 + 2x + 6$.

For the following function, describe the domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.

117. $f(x) = [[x]]$

Identify the change in the parent function that will produce the related function shown as a dashed line.

118. $f(x) = \sqrt{x}$



119. Graph $f(x) = \begin{cases} 10 - x & x < -15 \\ x^2 + 1 & -6 < x < 6 \\ \dfrac{4x + 9}{x} & 10 < x \end{cases}$

120. State the domain of $f \circ g$. Then find $f \circ g$, including any additional restrictions necessary on the domain of the composition.

$f(x) = \frac{2}{x}$

$g(x) = \sqrt{x - 2}$

121. Complete the square to find two functions f and g such that $h(x) = [f \circ g](x)$.

$h(x) = x^2 - 4x + 1$

122. Graph $f(x) = \frac{-6}{x}$ and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

Determine whether f has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

123. $f(x) = \frac{x - 1}{x + 7}$

124. Graph $f(x) = \frac{1}{3}(x - 7)^2 - 1$.

125. Describe the end behavior of the graph of $f(x) = x^3(x + 3)(-5x + 1)$ using limits.

126. State the number of possible real zeros and turning points of $g(x) = x^4 - 13x^2 + 36$. Then determine all of the real zeros by factoring.

Divide using long division.

127. $(6x^6 + 11x^5 - 20x^4 - 27x^3 - 9x^2 - 6x + 12) \div (-2x - 5)$

128. $\frac{4x^5 - 18x^4 + 18x^3 + 24x^2 - 34x - 2}{-2x^3 - 6x^2 - 4x + 6}$

Determine if the following binomials are factors of $f(x)$. Then list all of the factors.

129. $f(x) = x^5 + 2x^4 - 4x^3 + 12x^2 + 31x - 42$; $(x + 3)$, $(x - 1)$

130. Determine the equation whose roots are 4, –4, and –4.

131. Solve $5x^3 - 23x^2 + 29x - 15 = 0$.

Write a polynomial function of least degree with real coefficients in standard form that has the given zeros.

132. $-5 - \sqrt{7}$, $-5 + \sqrt{7}$, and $-7 - 6\boldsymbol{i}$

Use the graphs of f to describe the transformation that results in the graph of g. Then sketch the graphs of g and f.

133. $f(x) = \left(\frac{1}{5}\right)^{x}$; $g(x) = \left(\frac{1}{5}\right)^{x+2} + 4$

134. $f(x) = e^{x}$; $g(x) = -5e^{x+4} + 2$

135. Sketch and analyze the graph of $d(x) = 6^{-x} - 4$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

136. Use Newton's Law of Cooling, $y = ae^{-kt} + c$, to find the temperature of a substance as a function the time t in minutes that it has spent cooling off. Two samples of the substance were heated in a container of boiling water until their initial temperatures were both 100° C. The first sample will be cooled by being left out at a room temperature of 24° C, and the second sample of the substance will instead be cooled off in a refrigerator with an inside temperature of c = 4° C. The value of a will equal the *difference* between each sample's initial temperature and that sample's surrounding temperature, and the cooling constant of the substance is k = 0.12.

Find the first sample's temperature after it has cooled for 20 minutes. Then find the second sample's temperature after it has cooled for 10 minutes.

137. Find the amount of time required to double an amount at 5.84% if the interest is compounded continuously.