Algebra I - ANSWER KEY

Section 1 – Order of Operations

The order of operations in mathematics is a set of rules to follow to determine which operation to do first when there are different operations within a single problem. The order to perform combined operations is called the **PEMDAS** rule. A common mnemonic for **PEMDAS** is Please Excuse My Dear Aunt Sally.

- Always work on the calculations within *parenthesis* first (if any)
- Next, calculate the *exponents*
- Then, carry out *multiplication* or *division*, working from *left to right*
- Lastly, do *addition* and *subtraction*, working from *left to right*

Example 1

Evaluate $16 - 8 \div 2^2 + 14$.

$$16 - 8 \div 2^2 + 14 = 16 - 8 \div 4 + 14$$
$$= 16 - 2 + 14$$
$$= 14 + 14$$
$$= 28$$

Example 2

Evaluate each expression.

a. $4 \div 2 + 5(10 - 6)$

$$4 \div 2 + 5(10 - 6) = 4 \div 2 + 5(4)$$
$$= 2 + 5(4)$$
$$= 2 + 20$$
$$= 22$$

b. $6\left[32 - (2 + 3)^2\right]$

$$6\left[32 - (2 + 3)^2\right] = 6\left[32 - (5)^2\right]$$
$$= 6[32 - 25]$$
$$= 6[7]$$
$$= 42$$

Section 1 – Exercises

Evaluate and simplify each expression.

1. 3^5 $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ $\quad 9 \quad \cdot \quad 9 \quad \overset{1}{3}$ 81 $\boxed{243}$	2. $10 + 8^3 \div 16$ $10 + 8 \cdot 8 \cdot 8 \div 16$ $10 + 512 \div 16$ $10 + 32$ $\boxed{42}$	3. $(12-6) \cdot 5^2$ $6 \cdot 5^2$ $6 \cdot 25$ $\boxed{150}$
4. $18 \div 9 + 2 \cdot 6$ $2 + 2 \cdot 6$ $2 + 12$ $\boxed{14}$	5. $3[10 - (27 \div 9)]$ $3[10 - 3]$ $3[7]$ $\boxed{21}$	6. $4[(6^3 - 9) \div 23]$ $4[(216 - 9) \div 23]$ $4[207 \div 23]$ $4[9]$ $\boxed{36}$
7. $\dfrac{8 + 3^3}{12 - 7}$ $\dfrac{8 + 27}{12 - 7} = \dfrac{35}{5}$ $= \boxed{7}$	8. $3[4 - 8 + 4^2(2+5)]$ $3[4 - 8 + 16(2+5)]$ $3[4 - 8 + 16(7)]$ $3[4 - 8 + 112]$ $3[-4 + 112]$ $3[108]$ $\boxed{324}$	9. $25 + \left[(16 - 3 \cdot 5) + \dfrac{12+3}{5}\right]$ $25 + \left[(16 - 15) + \dfrac{12+3}{5}\right]$ $25 + \left[1 + \dfrac{15}{5}\right]$ $25 + [1 + 3]$ $25 + [4]$ $\boxed{29}$

To multiply fractions, multiply the numerators and multiply the denominators. If the numerators and denominators have common factors, you can simplify before you multiply by cross canceling.

Example 2

Find each product.

a. $\dfrac{2}{5} \cdot \dfrac{1}{3}$

$$\dfrac{2}{5} \cdot \dfrac{1}{3} = \dfrac{2 \cdot 1}{5 \cdot 3}$$

$$= \dfrac{2}{15}$$

b. $\dfrac{3}{5} \cdot 1\dfrac{1}{2}$

$$\dfrac{3}{5} \cdot 1\dfrac{1}{2} = \dfrac{3}{5} \cdot \dfrac{3}{2}$$

$$= \dfrac{3 \cdot 3}{5 \cdot 2}$$

$$= \dfrac{9}{10}$$

c. $\dfrac{1}{4} \cdot \dfrac{2}{9}$

$$\dfrac{1}{4} \cdot \dfrac{2}{9} = \dfrac{1}{\overset{}{\underset{2}{4}}} \cdot \dfrac{\overset{1}{2}}{9}$$

$$= \dfrac{1 \cdot 1}{2 \cdot 9} \text{ or } \dfrac{1}{18}$$

Section 2 – Exercises

Find each product. Write your answer in simplest form. (Leave as an improper fraction.)

13. $\dfrac{3}{5} \cdot \dfrac{5}{6} = \dfrac{\overset{1}{\cancel{3}}}{\cancel{5}} \cdot \dfrac{\cancel{5}}{\cancel{6}_2}$ $= \boxed{\dfrac{1}{2}}$	14. $\dfrac{11}{3} \cdot \dfrac{9}{44} = \dfrac{\overset{1}{\cancel{11}}}{\cancel{3}} \cdot \dfrac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{44}}}$ $= \boxed{\dfrac{3}{4}}$	15. $3\dfrac{1}{2} \cdot 1\dfrac{1}{2} = \dfrac{7}{2} \cdot \dfrac{3}{2}$ $= \boxed{\dfrac{21}{4}}$
16. $-\dfrac{2}{7} \cdot 4\dfrac{2}{3} = -\dfrac{2}{7} \cdot \dfrac{\overset{2}{\cancel{14}}}{3}$ $= \boxed{\dfrac{-4}{3}}$	17. $-\dfrac{1}{3} \cdot -7\dfrac{1}{2} = -\dfrac{1}{\cancel{3}} \cdot -\dfrac{\overset{5}{\cancel{15}}}{2}$ $= \boxed{\dfrac{5}{2}}$	18. $\dfrac{1}{4} \cdot -3\dfrac{5}{6} = \dfrac{1}{4} \cdot -\dfrac{23}{6}$ $= \boxed{-\dfrac{23}{24}}$

To divide one fraction by another, you multiply the first fraction by the reciprocal of the second fraction.

<u>Example 3</u>

Find each quotient.

a. $\frac{1}{3} \div \frac{1}{2}$

$$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1}$$
$$= \frac{2}{3}$$

b. $\frac{3}{8} \div \frac{2}{3}$

$$\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \cdot \frac{3}{2}$$
$$= \frac{9}{16}$$

c. $\frac{3}{4} \div 2\frac{1}{2}$

$$\frac{3}{4} \div 2\frac{1}{2} = \frac{3}{4} \div \frac{5}{2}$$
$$= \frac{3}{4} \cdot \frac{2}{5}$$
$$= \frac{6}{20} \text{ or } \frac{3}{10}$$

d. $-\frac{1}{5} \div \left(-\frac{3}{10}\right)$

$$-\frac{1}{5} \div \left(-\frac{3}{10}\right) = -\frac{1}{5} \cdot \left(-\frac{10}{3}\right)$$
$$= \frac{10}{15} \text{ or } \frac{2}{3}$$

Section 2 – Exercises

Find each quotient. Write your answer in simplest form. (Leave as an improper fraction.)

19. $\frac{3}{25} \div \frac{2}{15}$

$$\frac{3}{\underset{5}{\cancel{25}}} \cdot \frac{\overset{3}{\cancel{15}}}{2}$$

$$\boxed{\frac{9}{10}}$$

20. $2\frac{1}{4} \div \frac{1}{2}$

$$\frac{9}{\underset{2}{\cancel{4}}} \cdot \frac{\overset{1}{\cancel{2}}}{1}$$

$$\boxed{\frac{9}{2}}$$

21. $-\frac{9}{10} \div 3 \rightarrow -\frac{9}{10} \div \frac{3}{1}$

$$-\frac{\overset{3}{\cancel{9}}}{10} \cdot \frac{1}{\cancel{3}_1}$$

$$\boxed{-\frac{3}{10}}$$

Section 3 – Real Number Comparison

An inequality is a mathematical sentence that compares the value of two expressions using an inequality symbol.

Inequality Symbol	Pronounced	Example
<	Less than	$4 < 9$
≤	Less than or equal to	$-3 \leq 2$
>	Greater than	$-4 > -7$
≥	Greater than or equal to	$5 \geq 5$
≠	Not equal to	$7 \neq 11$

Example 1

Which one is greater, $\dfrac{4}{9}$ or $\dfrac{5}{12}$?

Rewrite each fraction using the LCD.

$$\frac{4}{9} = \frac{16}{36} \quad \text{and} \quad \frac{5}{12} = \frac{15}{36}$$

$$\frac{16}{36} > \frac{15}{36} \qquad \text{So} \quad \frac{4}{9} > \frac{5}{12}$$

Section 3 – Exercises

Use <, =, or > to compare the numbers.

22. $-12 \;\boxed{>}\; -15$

23. $0.63 \;\boxed{>}\; 0.6$

24. $0.88 \;\boxed{<}\; \dfrac{8}{9}$

25. $\dfrac{2}{3} \;\boxed{>}\; \dfrac{1}{6}$

26. $\dfrac{3}{4} \;\boxed{=}\; \dfrac{12}{16}$

27. $-2\dfrac{5}{8} \;\boxed{<}\; -2\dfrac{1}{2}$

Section 4 – Variables & Verbal Expressions

Translating in mathematics usually involves changing a verbal phrase into a mathematical phrase. The following are common phrases used in mathematics.

Phrase	Sign
sum, increased, added to, more than, plus, totals, combined, perimeter	+
difference, minus, less than, used, remain, subtracted from, decreased by	−
product, of, times, area, doubles, multiplied by	•
quotient, division, average, half, divided by, per	÷
is, is the same as, equal, was, were, has, costs, becomes	=

Section 4 – Exercises

Write an algebraic expression for each phrase.

28. 7 increased by x + $7 + x$	29. the difference of 8 and n − $8 - n$
30. the product of 2 and t • $2t$	31. 10 decreased by m − $10 - m$
32. 32 divided by d ÷ $32 \div d \Rightarrow \dfrac{32}{d}$	33. 12 less than p − flip order $p - 12$
34. the sum of 7 and h + $7 + h$	35. 9 plus the quotient of y and 15 ÷ $9 + \dfrac{y}{15}$

Section 5 – Evaluating Algebraic Expressions

A *variable* is a letter that represents an unspecified number. To evaluate an algebraic expression, replace the variables with their values. Then find the value of the numerical expression using the order of operations.

Example 1

Evaluate $3x^2 + (2y + z^3)$ if $x = 4$, $y = 5$, $z = 3$.

$3x^2 + (2y + z^3)$

$= 3(4)^2 + (2 \cdot 5 + 3^3)$

$= 3(4)^2 + (2 \cdot 5 + 27)$

$= 3(4)^2 + (10 + 27)$

$= 3(4)^2 + (37)$

$= 3(16) + 37$

$= 48 + 37$

$= 85$

Section 5 – Exercises

Evaluate each expression.

36. xy for $x = 3$, $y = 16$

xy

$(3)(16) = \boxed{48}$

37. $n + 2$ for $n = -7$

$n + 2$

$(-7) + 2 = \boxed{-5}$

38. $10 - r + 5$ for $r = 23$

$10 - (23) + 5$

$\underline{10 - 23 + 5}$

$-13 + 5 = \boxed{-8}$

39. $t + u \div 6$ for $t = 12$, $u = 18$

$t + u \div 6$

$12 + 18 \div 6$

$12 + 3 = \boxed{15}$

40. $4p - 26$ for $p = 10$

$4p - 26$

$4(10) - 26$

$40 - 26 = \boxed{14}$

41. $m^2 - 7$ for $m = 11$

$m^2 - 7$

$(11)^2 - 7$

$121 - 7 = \boxed{114}$

42. $3ab - c$ for $a = -4$, $b = 2$, $c = 5$

$3ab - c$

$3(-4)(2) - (5)$

$-24 - 5 = \boxed{-29}$

43. $\dfrac{ab}{2} - 4c$ for $a = 6$, $b = 5$, $c = 3$

$\dfrac{ab}{2} - 4c$

$\dfrac{(6)(5)}{2} - 4(3)$

$\dfrac{30}{2} - 12 = 15 - 12 = \boxed{3}$

Section 6 – Solving One-Step Equations

In an equation, the variable represents the number that satisfies the equation. To solve an equation means to find the value of the variable that makes the equation true. You will only need to perform one step in order to solve a **one-step** equation.

The strategy for getting the variable by itself involves using opposite operations. The most important thing to remember in solving a linear equation is that whatever you do to one side of the equation, you MUST do to the other side.

Example 1

$-2 = k - 14$	Solve
$-2 + 14 = k - 14 + 14$	Since 14 is subtracted from k, you must add 14 to **each** side of the equation
$12 = k$ or $k = 12$	Answer

Example 2

$\dfrac{x}{-7} = 15$	Solve
$(-7)\dfrac{x}{-7} = 15(-7)$	Since x is divided by -7, you must multiply **both** sides by -7
$x = -105$	Answer

Example 3

$\dfrac{3}{4}u = -24$	Solve
$\left(\dfrac{4}{3}\right)\dfrac{3}{4}u = -24\left(\dfrac{4}{3}\right)$	Multiply both sides by the reciprocal of $\dfrac{3}{4}$ and cancel any common factors
$u = -32$	Answer

Section 6 – Exercises

Solve each equation.

44. $37 = x - 72$ $$37 = x - 72$$ $$+72 \qquad +72$$ $$\overline{109 = x}$$ $$\boxed{x = 109}$$	45. $5p = 325$ $$\frac{5p}{5} = \frac{325}{5}$$ $$p = 65$$ $$\boxed{p = 65}$$
46. $d + 1.5 = 3.7$ $$d + 1.5 = 3.7$$ $$-1.5 \quad -1.5$$ $$\overline{d \qquad = 2.2}$$ $$\boxed{d = 2.2}$$	47. $102 + t = 36$ $$102 + t = 36$$ $$-102 \qquad -102$$ $$t = -66$$ $$\boxed{t = -66}$$
48. $\frac{2}{3}y = 8$ $$\left(\frac{3}{2}\right)\frac{2}{3}y = 8\left(\frac{3}{2}\right)$$ $$y = \frac{24}{2}$$ $$\boxed{y = 12}$$	49. $\frac{h}{7} = -12$ $$(7)\frac{h}{7} = -12(7)$$ $$\boxed{h = -84}$$
50. $\frac{3}{5}g = -6$ $$\left(\frac{5}{3}\right)\frac{3}{5}g = -6\left(\frac{5}{3}\right)$$ $$g = \frac{-30}{3}$$ $$\boxed{g = -10}$$	51. $\frac{1}{4}m = \frac{5}{8}$ $$\left(\frac{4}{1}\right)\frac{1}{4}m = \frac{5}{8}\left(\frac{4}{1}\right)$$ $$m = \frac{20}{8}$$ $$\boxed{m = \frac{5}{2}}$$

Section 7 – Measures of Central Tendency

In working with statistical data, it is often useful to determine a single quantity that best describes a set of data. The best quantity to choose is usually one of the most popular measures of central tendency: mean, median, mode, or range.

Mean   The mean is the sum of the data items in a set divided by the number of data items in the set.

Median   The median is the middle value in a set of data when the numbers are arranged in numerical order. If the set has an even number of data items, the median is the mean of the two middle data values.

Mode   The mode is the data item that occurs most often in a set of data.

Range   The range is the difference between the greatest and least values in a set of data.

Example 1

Set of data: 34, 46, 31, 40, 33, 40, 35

In order: 31, 33, 34, 35, 40, 40, 46

Mean	$\dfrac{(31+33+34+35+40+40+46)}{7}$	Answer: 37
Median	35 is the middle number when written in numerical order	Answer: 35
Mode	40 is the only number that occurs more than once	Answer: 40
Range	$46 - 31$	Answer: 15

Example 2

Set of data: 41, 28, 37, 56, 34, 61

In order: 28, 34, 37, 41, 56, 61

Mean	$\dfrac{(28+34+37+41+56+61)}{6}$	Answer: $42.8\overline{3}$
Median	$\dfrac{37+41}{2}$ (There are an even number of numbers in the data set)	Answer: 39
Mode	No number repeats more than once	Answer: None
Range	$61 - 28$	Answer: 33

Section 7 – Exercises

Find the mean, median, mode, and range of each set of data.

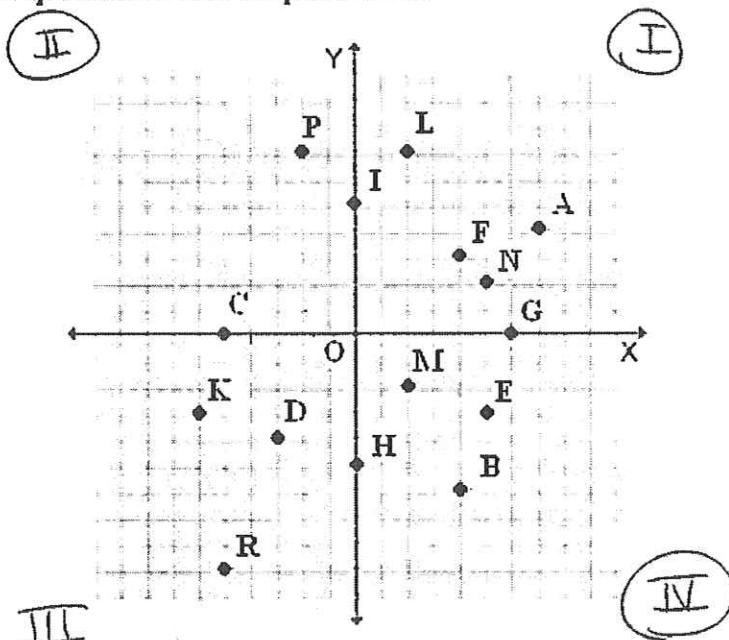|	
52. Daily sales from a store $834, $1099, $775, $900, $970 mean = $\dfrac{834 + 1099 + 775 + 900 + 970}{5}$ mean = 915.6 median: 775, 834, 900, 970, 1099 range = 1099 – 834 = 265 Mean = $915.60 Median = $900 Mode = no mode Range = 265	**53.** Goals scored in a soccer game 3, 2, 0, 11, 7, 6, 4, 10 mean = $\dfrac{3 + 2 + 0 + 11 + 7 + 6 + 4 + 10}{8}$ mean = 5.375 median: 0, 2, 3, 4, 6, 7, 10, 11 $\dfrac{4+6}{2}$ = ⑤ range = 11 – 0 = 11 Mean = 5.375 Median = 5 Mode = no mode Range = 11
54. Number of days above 50° in last 5 months 6, 8, 15, 22, 8 mean = $\dfrac{6 + 8 + 15 + 22 + 8}{5}$ mean = 11.8 median: 6, 8, 8, 15, 22 mode: 8 appears twice (the most) range = 22 – 6 = 16 Mean = 11.8 Median = 8 Mode = 8 Range = 16	**55.** Height of players on a basketball team (inches) 72, 74, 70, 77, 76, 72 mean = $\dfrac{72 + 74 + 70 + 77 + 76 + 72}{6}$ mean = 73.5 median: 70, 72, 72, 74, 76, 77 $\dfrac{72+74}{2}$ = 73 mode: 72 appears the most range: 76 – 70 = 6 Mean = 73.5 Median = 73 Mode = 72 Range = 6

Section 8 – Plotting on the Coordinate Plane

You can graph points on a coordinate plane. Use an ordered pair (*x*, *y*) to record the coordinates. The first number in the pair is the *x-coordinate*. The second number is the *y-coordinate*. To graph a point, start at the origin (0, 0). Move **horizontally** according to the value of *x*. Then move **vertically** according to the value of *y*.

Section 8 – Exercises

List the ordered pair for each letter, then identify the quadrant or axes the point lies in.

56. C	(-5,0)	no quadrant x-axis
57. A	(7,4)	quadrant I
58. M	(2,-2)	quadrant IV
59. P	(-2,7)	quadrant II
60. F	(4,3)	quadrant I
61. I	(0,5)	no quadrant y-axis
62. R	(-5,-9)	quadrant III
63. E	(5,-3)	quadrant IV

Plot & label the following ordered pairs.

64. F = (–8, 6)
65. R = (6, –1)
66. I = (–5, –7)
67. E = (4, 9)
68. N = (2, –3)
69. D = (–4, 0)
70. S = (0, 7)